THE UNCERTAINTIES CALCULATION OF ACOUSTIC METHOD FOR MEASUREMENT OF DISSIPATIVE PROPERTIES OF HETEROGENEOUS NON-METALLIC MATERIALS

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Abstract. The aim of this study is to prepare the budget for measurement uncertainty of dissipative properties of composite materials. The method used to study the vibrational energy dissipation characteristics based on coupling of vibrations damping decrement and acoustic velocity in a non-metallic heterogeneous material is reviewed. The proposed method allows finding the dependence of damping on vibrations amplitude and frequency of strain-stress state of material. The uncertainties calculation of acoustic method for measurement of dissipative properties of heterogeneous non-metallic materials is compiled.

Keywords: dissipative properties, damping decrement, an acoustic measurement method, measurement uncertainty.

Introduction. The main advantage of heterogeneous non-metallic materials over traditional...
consists in the increased vibrostability. The effective use of these materials and structures needs measurement of reliable values of dissipation characteristics, as well as common factors of their change during the loading process.

Various mechanisms and corresponding models for measurement of reliable values of traditional materials’ dissipation characteristics are exposed by such authors as: G.S. Pysarenko, N.N. Afanasyev, A.Yu. Ishlinsky, V.L. Biderman, V.V. Matveev, V.T. Troshchenko and others. The monographs by B.L. Pelech, B.I. Salyak, V.V. Bolotin, V.V. Vasyliev, Yu.N. Novychkov, A.P. Yakovlev, A.N. Guz, V.P. Tamuzh and others have been devoted to the elaboration of research methods applied to the damping in the non-metallic heterogeneous materials.

The heterogeneous non-metallic materials are characterized by complex processes of energy dissipation due to such factors [1]:

— Internal damping with viscid matrix and hard fillers;
— Structural dissipation on the “matrix – filler” distribution boundaries.

The aforementioned experimental methods used in the study of dissipative properties of heterogeneous non-metallic materials, characterized by complex energy dissipation processes, lead to significant errors and makes impossible to determine the effect of each separately existing energy dissipation mechanisms.

The acoustic method for measurement of dissipative properties of heterogeneous non-metallic materials allows finding the dependence of damping on vibrations amplitude and frequency of strain-stress state of material. Moreover, this method can be used not only for samples of the material but also for finished products. Although the method has several advantages, the problem of accuracy of the obtained results is remains relevant today.

The aim of this study is to prepare the budget for measurement uncertainty of dissipative properties of composite materials.

Materials and Methods. The method used to study the vibrational energy dissipation characteristics based on coupling of vibrations damping decrement and acoustic velocity in a non-metallic heterogeneous material. The dependence of the vibrations damping decrement on the material’s elastic modulus is described by the formula [1]:

$$\lambda = 2.076 \cdot 10^{-5} \cdot E^2 - 2.109 \cdot 10^{-3} \cdot E + 0.073,$$

where

$\lambda$ — vibrations damping decrement;

$E$ — material’s elastic modulus, GPa.

The propagation velocity of transverse acoustic wave over controlled environment, i.e. synthegran, can be found according to [2] as

$$C = \sqrt{\frac{E}{2\rho(1 + \nu)}},$$

where

$C$ — propagation velocity of transverse acoustic wave over synthegran, m/s;

$\rho$ — synthegran density, kg/m$^3$;

$\nu$ — Poisson’s ratio.

By substituting (2) into (1), we obtain the dependence of the vibrations damping decrement on the velocity of acoustic wave at synthegran:

$$\lambda = 8.304 \cdot 10^{-5} \rho^2 (1 + \nu)^2 C^2 - 4.218 \cdot 10^{-3} \rho(1 + \nu) C^2 + 0.073.$$

Figure 1 shows the scheme of the setup for measurement of the acoustic wave velocity.

A test sample attached to a console. On it, at a fixed distance $S$ (sound control base) is a set of the vibroacoustical piezoelectric sensors AVS-117 (all sensors are identical in size and weight) which receiving signals, which are proportional to the displacement. The sample is subjected to shock loading. The torque strike is implemented by the falling steel ball (diameter of 40 mm, drop height of 200 mm). The guide cylinder, which is set perpendicular to the controlled surface, provides the ball dropping to required point of surface.
The sensors’ signals are registered by the electronic diagnostic complex “Dolphin-1M”, which provides switching, coordination, pre-filtering and input of measured data to a computer via an analog-to-digital converter. Each channel frequency response is 280 kHz.

![Diagram of the setup for measurement of the acoustic wave velocity](image)

**Fig. 1. Scheme of the setup for measurement of the acoustic wave velocity**

The propagation velocity of acoustic wave in synthegran is determined by pulse method considering the time difference between signals of vibroacoustic sensors in accordance with the following formula:

\[
C = \frac{S}{\Delta t},
\]

where \( \Delta t = t_1 - t_2 \) — time difference between signals of vibroacoustic sensors, s;

\( t_1, t_2 \) — time of acoustic signals arrival to the first and the second sensors respectively, s.

The difference of damping values, which were found using different methods for the same sample, caused not only by different assumptions of the dissipative forces kind, but the dependence of the accuracy and sensitivity of used experimental methods on the measured value [3]. Such dependence involves an effective domain of the measurement method applicability, in which we can get the most accurate energy dissipation characteristics for a particular material or design element. The proposed measurement method of energy dissipation characteristics in heterogeneous non-metallic materials is indirect with a lot of the measured value changes from tested object to measurement results (Fig. 2), that, as known from [4], leads to the accumulation of measurement errors at each transformation phase.

Now we prepare the budget for indirect measurement uncertainty of damping decrement of synthegran sample. The application of the measurement uncertainty for measurements’ quality assessment is described in [5]. This approach includes an assessment of uncertainty:

— uncertainty type \( A \) — using methods of mathematical statistics for processing of the measurement results;

— uncertainty type \( B \) — using other methods based on using the information from technical standards.

Now we formulate the research task to identify the causes of errors.

We obtain next values by direct measurements: elastic modulus \( E = 25 \text{ GPa}, \) Poisson’s ratio \( \nu = 0.26, \) length of sound control base \( S = 263 \text{ mm}, \) time difference between signals of vibroacoustic sensors \( t = 132 \mu s. \) We used the samples of regular shape, so the material density was determined by the ratio of its mass to its volume \( \rho = 2500 \text{ kg/m}^3. \) The next values of damping decrement have been received: \( \lambda_1 = 0.027; \lambda_2 = 0.028; \lambda_3 = 0.027; \lambda_4 = 0.027; \lambda_5 = 0.027. \)

The total uncertainty \( u_C \) is calculated using the formula:

\[
u_C = \sqrt{u_A^2 + u_B^2},
\]

where \( u_A \) — uncertainty type \( A; \)

\( u_B \) — uncertainty type \( B. \)
The first group of errors (uncertainty type \(A\)) contains of parameters changing of calibrated bump in tolerance limits, displacement of sensor in repeated placement to measurement point, layer thickness variation of contact agent because of irregular hold-down of resolvers to control surface, inaccuracy in reading and etc.

As is well known, the random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations. In practice, to obtain a satisfactory error value at the lowest cost the sufficient repetitiveness is 5 (rarely 7) measurements at the point of control.

The second group of errors (uncertainty type \(B\)) is linked with density and Poisson’s ratio measurement errors, distance between sensors, time difference between signals of vibroacoustic sensors.

The standard uncertainty \(u_A\) of measurement of damping decrement in synthegran (mean-square deviation), when the measurement result is defined as the arithmetic mean, is calculated by the formula:

\[
u_A = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\lambda_i - \bar{\lambda})^2}, \tag{6}
\]

where \(n\) — number of measurements at one point of the controlled surface;
\(\lambda_i\) — \(i\)-th vibrations damping decrement value;
\(\bar{\lambda}\) — average value of vibrations damping decrement.

By substituting the measurement results, we obtain
\(\bar{\lambda} = 0.0272; \quad u_A = 0.0001\).

Consider the calculation of a standard uncertainty type \(B\). Such uncertainty is usually represented as a deviation of values magnitude from its assessment. The most common way to formalize incomplete knowledge of the value is to postulate the uniform distribution of the possible values between the given limits [5]. Thus, for symmetric limits a standard uncertainty is calculated using the formula

\[
u_B(x_i) = \frac{b_i}{\sqrt{3}}, \tag{7}
\]

where \(x_i\) — assessment of \(i\)-th input value;
\(b_i\) — symmetric limits of deviation of the measured value from the measurements.
The total standard uncertainty type $B$ is calculated using the formula

$$u_B = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2_B(x_i)}.$$  \hfill (8)

The propagation velocity of elastic waves is determined by the formula (2). There were five measurements, and the calculated arithmetic mean, which amounted to $C = 1879$ m/s.

The standard uncertainty $u_{Bl}$ of measurement of sound control base length by calipers is $u_{Bl} = \pm 0.58$ mm.

The relative uncertainty $\delta l$ was calculated using the formula

$$\delta l = \frac{\Delta l}{l} \cdot 100 \% = 0.02 \%.$$  \hfill (9)

The phase shift of “Dolphin-1M” complex is less than 1 degree per Hz, and such value can be ignored. Moreover, given that the velocity of elastic waves is determined by the wave-front, and its waveshape does not affect measurement accuracy. Therefore, the uncertainty of time measurement can be determined by quantization error only. Thus, the limits of deviation of the measured value from the measurements can never exceed half of the quantization step size (when rounded to the nearest integer). The time quantization step is $q = 3.5 \mu$s. Therefore,

$$\Delta q = \frac{q}{2} = \frac{3.5}{2} = \pm 1.75 \mu s.$$  \hfill (10)

According to the formula (8), the standard uncertainty of measurement of time is

$$u_{Bl} = \pm 1.01 \mu s;$$

and in relative form

$$\delta t = 0.77 \%.$$ 

Now we find the uncertainty of indirect measurement of density. When the uncertainty of mass measurement is $\pm 0.3$ g and the uncertainty of sample linear dimensions measurement is $\pm 0.6$ mm (by vernier caliper) the relative uncertainty can never exceed 1.2 %. So, the standard uncertainty of indirect density measurement will be calculated as follows

$$u_{B\rho} = \frac{2500 \cdot 1.2}{100} = \pm 29 \, \text{kg/m}^3.$$ 

According to the ratings of extensometer “EEO” the relative uncertainty of measurement of Poisson’s ration is 0.23 %; in absolute terms — $u_{B\nu} = \pm 0.0006$.

Given (8), the uncertainty of indirect measurement of acoustic wave’s velocity in synthegran will be calculated as follows

$$u_{Bw} = \sqrt{(-u_{Bl})^2 + (u_{Bl})^2},$$  \hfill (11)

$$u_{Bw} = 14 \, \text{m/s};$$

the relative uncertainty of such measurement can never exceed 0.7 %.

From (2), we obtain

$$E = 2\rho C^2 (1+\nu).$$

According to the formula (8), the standard uncertainty of indirect measurement of elastic modulus will be calculated as follows

$$u_{BE} = \sqrt{\left( \frac{\partial E}{\partial \rho} \right)^2 u^2_{B\rho} + \left( \frac{\partial E}{\partial \nu} \right)^2 u^2_{B\nu} + \left( \frac{\partial E}{\partial C} \right)^2 u^2_{Bw}}.$$  \hfill (12)
If we take the partial derivatives, we obtain
\[ \Delta E = \sqrt{\left(2C^2[1 + \nu]\right)^2 u_{B_0}^2 + (2C^2\rho)^2 u_{B_e}^2 + (4C\rho[1 + \nu])^2 u_{B_{ne}}^2}. \] (13)

Finally, the result is
\[ u_{BE} = \pm132 \cdot 10^4 \approx \pm0.1 \text{ GPa}. \]

Using (9), we calculate the relative uncertainty of measurement of elastic modulus
\[ \delta E = \frac{0.1}{25} \times 100\% = 0.4\%. \]

Now we find the uncertainty of indirect measurement of damping decrement. According to the formula (8), the standard uncertainty of indirect measurement of elastic modulus is calculated as follows:
\[ u_{B_0} = \sqrt{\left(\frac{\partial \lambda}{\partial E}\right)^2 u_{BE}^2}. \] (14)

If we take the partial derivatives, we obtain
\[ u_{B_0} = \sqrt{(0.0004152E + 2.109 \cdot 10^{-3})^2 u_{BE}^2}. \] (15)

Substituting values, we obtain
\[ u_{B_0} = \pm0.001. \]

In relative form the systematic error of measurement method is
\[ \delta \lambda = \pm3.7\%. \]

Now we find the total standard uncertainty. Given the standard uncertainty type \( A \) is 10-times less than the standard uncertainty type \( B \), and then it can be ignored. So
\[ u_T = u_{B_0} = \pm0.001. \]

In practical terms, when calculating the uncertainty of measurement of damping decrement, the distribution of the measured value is accepted as uniform. Such a suggestion is preferred in cases where the uniformity of the values distribution is not proven.

The expanded uncertainty \( u_\rho \) for confidence level \( P = 0.95 \) can be represented as
\[ u_{0.95} = ku_T, \] (16)

where \( k \) — coverage factor, which depends on the confidence level \( P \) and the number of degrees of freedom \( v_{eff} \), which is defined by the formula
\[ v_{eff} = \frac{u_T^2}{\sum_{i=1}^{m} \frac{u_i^4(x_i)}{v_i \left( \frac{df}{dx_i} \right)}}. \] (17)

where \( v_i = n_i - 1 \) in the case of uncertainty type \( A \); \( v_i = \infty \) in the case of uncertainty type \( B \).

Given (17) for the considered case, we obtain
\[ v_{eff} = \frac{u_T^2}{\sum_{i=1}^{m} \frac{u_i^4(x_i)}{v_i \left( \frac{df}{dx_i} \right)}} = 4 \cdot 10^{16}. \] (18)

With this number of degrees of freedom and confidence level \( P = 0.95 \) the coverage factor \( k \) is equal to 1.96 [5]. Then, considering (16), the expanded uncertainty is
So, the final result of measurement of vibrations damping decrement at synthegran can be represented as
\[
\lambda = 0.0272 \pm 0.00196,
\]

at \( P = 0.95 \) and uniform distribution law.

**Results and Discussion.** The method used to study the vibrational energy dissipation characteristics based on coupling of vibrations damping decrement and acoustic velocity in a non-metallic heterogeneous material is reviewed. The international approach for evaluation of measurements quality is used. It includes the common practice international rules for uncertainty expression and their summation. These rules are used as internationally acknowledged confidence measure to the measurement results, which includes testing. Based on analysis of the measurement errors reasons the uncertainties budgeting of acoustic method for measurement of dissipative properties of heterogeneous non-metallic materials were compiled. It was defined that there are two groups of reasons resulting in errors during measurement of heterogeneous non-metallic materials dissipative properties. The first group of errors contains of parameters changing of calibrated bump in tolerance limits, displacement of sensor in repeated placement to measurement point, layer thickness variation of contact agent because of irregular hold-down of resolvers to control surface, inaccuracy in reading and etc. The second group of errors is linked with density and Poisson’s ratio measurement errors, distance between sensors, time difference between signals of vibroacoustic sensors. The expanded uncertainty is \( u_{0.95} = 0.00196 \) at confidence level \( P = 0.95 \) and uniform distribution law.

**References**