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SOLUTION OF THE SPECIFIC BOUNDARY PROBLEM IN THE CASE OF THE ONE-DIMENSIONAL WAVEGUIDE

І.Ю. Дмитрієва. **Розв'язання спеціальної крайової задачі для одномірного хвилеводу.** Запропоновано аналітичне розв'язання крайової задачі, яка породжена симетричною системою диференціальних рівнянь Максвела у випадку довільно збудженого лінійного ізотропного однорідного середовища. Початкові та граничні умови розглядаються для одномірного пів нескінченного хвилеводу, вважаючи основною змінною координату часу. Одержані результати дозволяють розв'язувати у явному вигляді задачі розповсюджування сигналів у вказаних середовищах, в тому числі і питання мобільного зв'язку.
Ключові слова: крайова задача, однорідне ізотропне лінійне довільно збуджене середовище, одновимірний півнескінченний хвилевод.

И.Ю. Дмитриева. **Решение специальной краевой задачи для одномерного волновода.** Предложено аналитическое решение краевой задачи, порожденной симметричной системой дифференциальных уравнений Максвелла в случае произвольно возбужденной линейной изотропной однородной среды. Начальные и граничные условия рассматриваются для одномерного полу- бесконечного волновода, считая основной переменной координату времени. Полученные результаты позволяют решать в явном виде задачи распространения сигналов в указанных средах, в том числе и вопросы мобильной связи.
Ключевые слова: краевая задача, однородная изотропная линейная произвольно возбужденная среда, одномерный полубесконечный волновод.

I.Yu. Dmitrieva. **Solution of the specific boundary problem in the case of the one-dimensional waveguide.** We propose an analytic solution of the boundary problem that is generated by the symmetrical system of the differential Maxwell equations in the case of an arbitrary excited linear isotropic homogeneous medium. The initial and boundary conditions are given for the one-dimensional semi-infinite waveguide, considering the temporal coordinate as the main variable. The present results allow solving explicitly the problems of the signal transmission in the aforesaid media including the questions of the mobile communications.
Keywords: the boundary problem, homogeneous isotropic linear arbitrary excited medium, the one-dimensional semi-infinite waveguide.

It is well known that the signal transmissions in the various kinds of media can be described analytically by means of the corresponding systems of PDEs [1, 2]. In its turn, these mathematical objects generate the respective boundary problems that reflect the aforesaid physical phenomena in the framework of the appropriate temporal and spatial restrictions [3, 4]. Solving such problems explicitly, one covers simultaneously two of the researching directions, as of the classical electromagnetic field theory, as of the current technical electrodynamics. Both of them are tightly connected and equally important.

Classical electrodynamics is the fundamental basis of technical radio electronics and the modern theory of electromagnetic wave propagation too [5].

Hence, an explicit analytical study of the relevant vector boundary problems remains essentially necessary even nowadays, when a lot of applied computer programs exist. Moreover, if one investigates the concrete industrial or physical phenomenon, it is quite natural to construct first of all its mathematical model whose correct analytic solution is in conformity with the original problem statement and does not infringe it.

The preferable approach here is the strict but simple mathematical procedure that can be understood by the majority of engineers, staying at the same time in the limits of the usual program of higher mathematics which is lectured at the technical universities.

That is why we propose the essential industrial problem whose explicit solution bases on the general analytical diagonalization method. The latter represents the operator analogy of the classical algebraic Gauss algorithm [6], and was applied successfully when the differential symmetrical Maxwell system was studied [7]. Such system describes the electromagnetic wave propagation in the various kinds of media and can be used in the mobile connections as well.

The virtue of the above mentioned operator diagonalization approach is its independence, either on the original matrix structure, or on the initial and boundary conditions. So, it is very convenient to reduce the given system to the equivalent totality of the respective scalar equations, and only afterwards to formulate the relevant boundary problem.

Further, if we compare even the most advanced mathematical applied achievements studying the modern electromagnetic field problems, it can be noticed that almost all of them originally are bound to the initial and boundary conditions [8]. Moreover, the proposed solution relies basically on the concrete spatial coordinates, not on the temporal one.

Finally, it is easier to formulate and study the scalar boundary problem for an only one function than deal with the vector analogy that includes even the finite system of the corresponding equations.

Therefore, the *main purpose of the present paper* is the analytic solution of the specific boundary scalar problem that is generated by the symmetrical differential Maxwell system in the case of an arbitrary excited linear isotropic homogeneous medium. The initial and boundary conditions are given for the one-dimensional semi-infinite waveguide considering the temporal coordinate as the main variable.

Such problem statement in the scalar case is obviously approved owing to [9] where the solvability criterion of the symmetrical differential Maxwell system is proved. The medium is the same as we consider here, and the unified scalar wave PDE represents the equivalent object instead of the original Maxwell system.

Before we come directly to the aforesaid scalar problem study, it should be noted that the suggested formulation is really specific, since there is no one-dimensional analogy of the classical **rot** (**rotor**) operation. Therefore, we can not reduce formally the unified wave PDE [9] from the statement of (x, y, z, t) to the case of (x, t). It means that just in the present situation the complete diagonalization process of the original Maxwell system must be done as the first step, and the unified wave PDE is its final result. Only after that we have right to formulate the required boundary problem basing on [9] and solving it explicitly by means of the integral transforms technique [10].

So, let the symmetrical differential Maxwell system be given in the case of an arbitrary excited linear isotropic homogeneous medium over the space (x,t), where x,t are the spatial and temporal variables respectively:

$$\begin{cases} \partial_1 \vec{\mathbf{H}} = (\sigma \pm \lambda\varepsilon_a)\vec{\mathbf{E}} + \varepsilon_a \dfrac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{j}}^{CT}, \\ -\partial_1 \vec{\mathbf{E}} = (r \pm \lambda\mu_a)\vec{\mathbf{H}} + \mu_a \dfrac{\partial \vec{\mathbf{H}}}{\partial t} + \vec{\mathbf{e}}^{CT}, \end{cases} \tag{1}$$

where:

$\vec{\mathbf{E}} = \vec{\mathbf{E}}(x,t)$, $\vec{\mathbf{H}} = \vec{\mathbf{H}}(x,t)$ — are the sought for vector functions of the electric and magnetic field tensions;

$\sigma, \mu_a, \varepsilon_a = \text{const} > 0$ — are the specific conductivity, absolute magnetic and dielectric permeability of the medium;

$\lambda = \text{const} > 0$ — is the parameter of the signal that excites the medium. An absorption of the signal by the medium corresponds to "+", a seizure implies "–";

$r>0$ — is the theoretical constant which is responsible for the "symmetry" of the right parts of (1) and whose existence is only assumed at the current stage of research;

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$\vec{\mathbf{j}}^{CT} = \vec{\mathbf{j}}^{CT}(x,t)$; $\vec{\mathbf{e}}^{CT} = \vec{\mathbf{e}}^{CT}(x,t)$ — are the known vector functions that describe the outside current sources and tensions;

$\partial_1 = \dfrac{\partial}{\partial x}$ — is the partial differential operation by the spatial variable x.

Vector functions $\vec{\mathbf{E}}$, $\vec{\mathrm{H}}$, $\vec{\mathrm{j}}^{CT}$, $\vec{\mathrm{e}}^{CT}$ belong to one and the same class of four-times continuously differentiated functions over the space (x,t) [9].

After simple algebraic transformations system (1) can be rewritten as follows

$$\begin{cases} -C\overrightarrow{\mathbf{F_1}} + \partial_1 \overrightarrow{\mathbf{F_2}} = \overrightarrow{\mathbf{f_1}}, \\ -\partial_1 \overrightarrow{\mathbf{F_1}} - D\overrightarrow{\mathbf{F_2}} = \overrightarrow{\mathbf{f_2}}, \end{cases} \tag{2}$$

where:

$$C = \sigma + \varepsilon_a \partial_0^*;\ D = r + \mu_a \partial_0^*;\ \partial_0^* = \partial_0 \pm \lambda;\ \partial_0 = \frac{\partial}{\partial t}; \tag{3}$$

are the symbols of the partial differential operators;

$$\overrightarrow{\mathrm{F}_1} = \vec{\mathrm{E}};\ \overrightarrow{\mathrm{F}_2} = \vec{\mathrm{H}};\ \overrightarrow{\mathrm{f}_1} = \vec{\mathrm{j}}^{CT};\ \overrightarrow{\mathrm{f}_2} = \vec{\mathrm{e}}^{CT} \tag{4}$$

are the unknown and given corresponding vector functions.

Further, (2) can be diagonalized either by the operator analogy of Gauss method [6, 7], or by the inverse matrix operator construction with respect to the original matrix (2). The last method is reflected in [9].

Namely, matrix from (2) looks like

$$\mathbf{M} = \begin{bmatrix} -C & \partial_1 \\ -\partial_1 & -D \end{bmatrix} \tag{5}$$

and

$$\det \mathbf{M} = \partial_1^2 + CD;\quad \mathbf{M}^{-1} = (\det \mathbf{M})^{-1} \begin{bmatrix} -D & -\partial_1 \\ \partial_1 & -C \end{bmatrix}, \tag{6}$$

where:

$\mathbf{M}^{-1}$ — is the inverse matrix operator regarding (5);

$\det \mathbf{M}$ — is the determinant of (5);

$(\det \mathrm{M})^{-1}$ – is the inverse operator of $\det \mathbf{M}$.

An application of (6) to (2) … (4) takes (2) to the unified wave PDE

$$\left(\tilde{\partial}_0^2 + \partial_1^2\right)\vec{\mathbf{F}} = \vec{\mathbf{f}}^*, \tag{7}$$

where:

$$\tilde{\partial}_0^2 = CD = (\sigma + \varepsilon_a \partial_0^*)(r + \mu_a \partial_0^*) = \varepsilon_a \mu_a \left(\partial_0^*\right)^2 + \left(\sigma\mu_a + r\varepsilon_a\right)\partial_0^* + \sigma r \tag{8}$$

is the operator differential polynomial;

$$\vec{\mathbf{F}} = \begin{bmatrix} \overrightarrow{\mathbf{F_1}} \\ \overrightarrow{\mathbf{F_2}} \end{bmatrix},\ \vec{\mathbf{f}}^* = \begin{bmatrix} -D\vec{\mathbf{f}}_1 - \partial_1 \vec{\mathbf{f}}_2 \\ \partial_1 \vec{\mathbf{f}}_1 - C\vec{\mathbf{f}}_2 \end{bmatrix} \tag{9}$$

are the sought for and known functions respectively.

It should be noted here that since the initial system (1) is considered for the space (x,t), then vector functions from (7), (9) can be accepted simultaneously as scalar in the framework, as of the original problem statement, as of the PDE (7).

Moreover, we have to repeat again that there is no right to reduce directly the unified general wave PDE from [6, 7] over the space (x,y,z,t) to the present case of (x,t). This fact is completely explicable because of the general situation [6, 7] when instead of ∂_1^2 from (7) the operator $\mathbf{rot}^2 = \mathbf{rot}\,\mathbf{rot}$ appears and acts. It is well known that the one-dimensional **rot** operator does not exist.

Now, taking into account the results of [9], we can formulate the wanted boundary problem for the semi-infinite axis over the space (x,t).

The mentioned statement is the following

$$\begin{cases} \left(\tilde{\partial}_0^2 + \partial_1^2\right)\vec{\mathbf{F}} = \vec{\mathbf{f}}^*, \;\; x,t \in [0,+\infty); \\ \vec{\mathbf{F}}(x,\mathbf{t})\big|_{t=0} = \vec{\mathbf{g}}_1(x); \\ \dfrac{\partial \vec{\mathbf{F}}(x,t)}{\partial t}\Big|_{t=0} = \vec{\mathbf{g}}_2(x); \\ \vec{\mathbf{F}}(0,t) = \vec{\mathbf{g}}_3(t) = \text{const} = d; \\ \dfrac{\partial^k \vec{\mathbf{F}}(x,t)}{\partial x^k}\Big|_{x=\infty} = 0,\; (k=0,1). \end{cases} \tag{10}$$

The given mathematical model (10) describes the wave propagation (signal transmission) in the semi-infinite one-dimensional waveguide along the direction of its spatial variable x and taking into account the temporal parameter t. The medium remains the same, as in (1), — isotropic linear homogeneous and an arbitrary excited. It is quite natural, since (10) is equivalent to (1) because of [9].

The first and the second initial conditions from (10) determine the behavior of the sought for vector field function and its instantaneous velocity at the zero time. The third, boundary condition from (10) concerns the behavior of the same function in terms of the spatial origin point and describes the signal independence of time remaining constant. The last, fourth boundary condition in (10) at the infinite point $(x=\infty)$ is required by the original problem statement and classical integral transform that must be applied here [10]. Physically it means the natural signal (wave) fading by its spatial coordinate x at the infinity.

The present contour of integration implies an application of the simplest version of the integral transforms. It is the continuous sine (sin):

$$\int_0^\infty \sin \alpha x \,\mathrm{d}x, \;\; \left(\alpha = const \in \mathbf{R} \setminus \{0\}\right). \tag{11}$$

The simplified geometrical picture of (10) is the following:

$$0x:\; 0\,\_\_\_\_\_\_\_\_\_\_\_\_\_\_\,(x,t)\,\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \to \infty$$

Applying (11) to (10) we get the equivalent problem in terms of the respective transforms, and it is written further. Only, it should be noted that the double integration by parts gives the result which we use here. Namely,

$$\int_0^\infty \partial_1^2 \vec{\mathbf{F}}(x,t) \sin \alpha x \,\mathrm{d}x = \alpha \vec{\mathbf{F}}(0,t) - \alpha^2 \vec{\mathbf{F}}_\alpha(t), \tag{12}$$

where:

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$\vec{\mathbf{F}}_\alpha(t) = \int_0^\infty \vec{\mathbf{F}}(x,t)\sin\alpha x\,dx$ — is the continuous sine-transform of $\vec{\mathbf{F}}(x,t)$ by the variable x.

Hence, using (11) for (10) and considering (12), we obtain the transform analogy of (10) with the appropriate ODE where $\vec{\mathbf{F}}_\alpha(t)$ is the unknown:

$$\begin{cases} \left(\dfrac{\mathrm{d}^2}{\mathrm{d}t^2} + \left(\dfrac{\sigma}{\varepsilon_a} + \dfrac{r}{\mu_a} \pm 2\lambda\right)\dfrac{\mathrm{d}}{\mathrm{d}t} + \left(\lambda^2 \pm \lambda\left(\dfrac{\sigma}{\varepsilon_a} + \dfrac{r}{\mu_a}\right) + \dfrac{r\sigma - \alpha^2 + \alpha d}{\mu_a\varepsilon_a}\right)\right)\vec{\mathbf{F}}_\alpha(t) = \vec{\mathbf{f}}^*_\alpha(t), \ t \in [0, +\infty); \\ \vec{\mathbf{F}}_\alpha(0) = \vec{\mathbf{g}}_{1\alpha}; \\ \dfrac{\mathrm{d}\vec{\mathbf{F}}_\alpha(0)}{\mathrm{d}t} = \vec{\mathbf{g}}_{2\alpha}; \\ \vec{\mathbf{g}}_{i\alpha} = \int_0^\infty \vec{\mathbf{g}}_i(x)\sin\alpha x\,\mathrm{d}x \ \ (i = 1,2). \end{cases} \tag{13}$$

In general, the second-order inhomogeneous linear ODE with the constant coefficients from (13) can be rewritten as follows

$$\vec{\mathbf{F}}_\alpha''(t) + a\vec{\mathbf{F}}_\alpha'(t) + b\vec{\mathbf{F}}_\alpha(t) = \vec{\mathbf{f}}^*_\alpha(t), \tag{14}$$

where:

$$\begin{aligned} a &= \frac{\sigma}{\varepsilon_a} + \frac{r}{\mu_a} \pm 2\lambda = const \in \mathbf{R}, \\ b &= \lambda^2 \pm \lambda\left(\frac{\sigma}{\varepsilon_a} + \frac{r}{\mu_a}\right) + \frac{r\sigma - \alpha^2 + \alpha d}{\mu_a\varepsilon_a} = const \in \mathbf{R} \end{aligned} \tag{15}$$

are its coefficients;

$$\vec{\mathbf{f}}^*_\alpha(t) = \int_0^\infty \vec{\mathbf{f}}^*_\alpha(x,t)\sin\alpha x\,\mathrm{d}t \tag{16}$$

is the transform of the function $\vec{\mathbf{f}}^*_\alpha(x,t)$.

Additionally, it is clear that (14)...(16) is the equation of forced oscillations [11] whose general solution is the sum of the particular solution of (14) and the general one of the corresponding homogeneous ODE with respect to (14). The latter is the equation of free oscillations [11], and its general solution is well-known [11]:

$$\vec{\mathbf{F}}_{\alpha 0}(t) = \begin{cases} C_1\exp\left(\dfrac{-a+D}{2}\right)t + C_2\exp\left(\dfrac{-a-D}{2}\right)t, \quad \text{if } D^2 > 0; \\ \exp\left(-\dfrac{at}{2}\right)\left(C_1\cos\dfrac{Dt}{2} + C_2\sin\dfrac{Dt}{2}\right) = C_3\exp\left(-\dfrac{at}{2}\right)\sin\dfrac{D(t-C_4)}{2}, \quad \text{if } D^2 < 0; \\ \exp\left(-\dfrac{at}{2}\right)(C_1 t + C_2), \quad \text{if } D^2 = 0, \end{cases} \tag{17}$$

where:

$$D^2 = a^2 - 4b = \left(\frac{\sigma}{\varepsilon_a} - \frac{r}{\mu_a}\right)^2 + \frac{4\alpha(\alpha - d)}{\mu_a\varepsilon_a} = const \in \mathbf{R}. \tag{18}$$

The aforesaid respective partial solution of (14) is the following [11]:

$$\vec{\mathbf{F}}_{\alpha 1}(t)=\begin{cases}\dfrac{2}{D}\int\limits_C^t \vec{\mathbf{f}}_\alpha^*(\tau)\exp\dfrac{a(\tau-t)}{2}\operatorname{sh}\dfrac{D(t-\tau)}{2}\mathrm{d}\tau, & \text{if } D^2>0;\\ \dfrac{2}{D}\int\limits_C^t \vec{\mathbf{f}}_\alpha^*(\tau)\exp\dfrac{a(\tau-t)}{2}\sin\dfrac{D(t-\tau)}{2}\mathrm{d}\tau, & \text{if } D^2<0;\\ \int\limits_C^t \vec{\mathbf{f}}_\alpha^*(\tau)(t-\tau)\exp\dfrac{a(\tau-t)}{2}\mathrm{d}\tau, & \text{if } D^2=0.\end{cases} \tag{19}$$

Then the required general solution of (14) is the sum of (17) and (19), where an addition is done in the appropriate cases

$$\vec{\mathbf{F}}_\alpha(t)=\vec{\mathbf{F}}_{\alpha 0}(t)+\vec{\mathbf{F}}_{\alpha 1}(t). \tag{20}$$

Realizing the initial conditions from (13) and taking into account (17)…(20) we get the system regarding the unknown real constants C_1, C_2 from (17):

$$\vec{\mathbf{F}}_\alpha(0)=\vec{\mathbf{F}}_{\alpha 0}(0)+\vec{\mathbf{F}}_{\alpha 1}(0)=\begin{cases}C_1+C_2-\dfrac{2}{D}\int\limits_C^0 \vec{\mathbf{f}}_\alpha^*(\tau)\exp\dfrac{a\tau}{2}\operatorname{sh}\dfrac{D\tau}{2}\mathrm{d}\tau, & \text{if } D^2>0\\ C_1-\dfrac{2}{D}\int\limits_C^0 \vec{\mathbf{f}}_\alpha^*(\tau)\exp\dfrac{a\tau}{2}\sin\dfrac{D\tau}{2}\mathrm{d}\tau, & \text{if } D^2<0\\ C_2-\int\limits_C^0 \vec{\mathbf{f}}_\alpha^*(\tau)\tau\exp\dfrac{a\tau}{2}\mathrm{d}\tau, & \text{if } D^2=0\end{cases}=\vec{\mathbf{g}}_{1\alpha}, \tag{21}$$

$$\vec{\mathbf{F}}'_\alpha(0)=\vec{\mathbf{F}}'_{\alpha 0}(0)+\vec{\mathbf{F}}'_{\alpha 1}(0)=\begin{cases}\dfrac{-a+D}{2}C_1-\dfrac{a+D}{2}C_2+\vec{\mathbf{F}}'_{\alpha 1}(0), & \text{if } D^2>0\\ -\dfrac{a}{2}C_1+\dfrac{D}{2}C_2+\vec{\mathbf{F}}'_{\alpha 1}(0), & \text{if } D^2<0\\ C_1-\dfrac{a}{2}C_2+\vec{\mathbf{F}}'_{\alpha 1}(0), & \text{if } D^2=0\end{cases}=\vec{\mathbf{g}}_{2\alpha}. \tag{22}$$

Simplifying (21), (22) with respect to the unknown constants C_1, C_2 we obtain the three linear systems:

$$\begin{cases}C_1+C_2=\vec{\mathbf{F}}_{\alpha 1}(0)+\vec{\mathbf{g}}_{1\alpha},\\ \dfrac{-a+D}{2}C_1-\dfrac{a+D}{2}C_2=-\vec{\mathbf{F}}'_{\alpha 1}(0)+\vec{\mathbf{g}}_{2\alpha}, \text{if } D^2>0;\end{cases} \tag{23}$$

$$\begin{cases}C_1=\vec{\mathbf{F}}_{\alpha 1}(0)+\vec{\mathbf{g}}_{1\alpha},\\ -\dfrac{a}{2}C_1+\dfrac{D}{2}C_2=-\vec{\mathbf{F}}'_{\alpha 1}(0)+\vec{\mathbf{g}}_{2\alpha}, \text{if } D^2<0;\end{cases} \tag{24}$$

$$\begin{cases}C_2=\vec{\mathbf{F}}_{\alpha 1}(0)+\vec{\mathbf{g}}_{1\alpha},\\ C_1-\dfrac{a}{2}C_2=-\vec{\mathbf{F}}'_{\alpha 1}(0)+\vec{\mathbf{g}}_{2\alpha}, \text{if } D^2=0,\end{cases} \tag{25}$$

where:

$\vec{\mathbf{g}}_{i\alpha}$ $(i=1,2)$ — are the same in all cases and are given in (13), (21), (22);

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$\vec{\mathbf{F}}_{\alpha 1}(0)$, $\vec{\mathbf{F}}'_{\alpha 1}(0)$ — are described in (19), (21), (22) and have their own different expressions in each of three values of D^2 from (18).

The corresponding solutions of (23)...(25) are given below:

$$C_{1,2} = \pm\frac{1}{D}\left(\vec{\mathbf{g}}_{2\alpha} - \vec{\mathbf{F}}'_{\alpha 1}(0)\right) + \frac{1}{2}\left(1 \pm \frac{a}{D}\right)\left(\vec{\mathbf{F}}_{\alpha 1}(0) + \vec{\mathbf{g}}_{1\alpha}\right), \tag{26}$$

$$\begin{cases} C_1 = \vec{\mathbf{F}}_{\alpha 1}(0) + \vec{\mathbf{g}}_{1\alpha}; \\ C_2 = \frac{1}{D}\left(a\left(\vec{\mathbf{F}}_{\alpha 1}(0) + \vec{\mathbf{g}}_{1\alpha}\right) + 2\left(-\vec{\mathbf{F}}'_{\alpha 1}(0) + \vec{\mathbf{g}}_{2\alpha},\right)\right), \end{cases} \tag{27}$$

$$\begin{cases} C_2 = \vec{\mathbf{F}}_{\alpha 1}(0) + \vec{\mathbf{g}}_{1\alpha}; \\ C_1 = -\vec{\mathbf{F}}'_{\alpha 1}(0) + \vec{\mathbf{g}}_{2\alpha} + \frac{a}{2}\left(\vec{\mathbf{F}}_{\alpha 1}(0) + \vec{\mathbf{g}}_{1\alpha}\right). \end{cases} \tag{28}$$

Substituting (26) – (28) for $C_{1,2}$ into (17), (20) we get the required analytic solution of (13).

Afterwards, an application of the inverse integral sine-transform

$$\vec{\mathbf{F}}(x,t) = \frac{2}{\pi}\int_0^\infty \vec{\mathbf{F}}_\alpha(t)\sin\alpha x \, d\alpha$$

to (20) gives at last the explicit solution of the original boundary problem (10).

Hence, the raised problem is solved analytically completely, and the purpose of the present paper is achieved here too.

References

1. Maxwell, J.C. Electromagnetic field theory / J.C. Maxwell // Transactions of the Cambridge Philosophical Society. — Cambridge, 1855 — 1856. — Vol. X. — 83 p.
2. Maxwell, J.C. Electrodynamics field theory / J.C. Maxwell // Royal Society Transactions. — London, 1864. — Vol. CLV. — 74 p.
3. Tikhonov, A.N. Uravneniya matematicheskoy fiziki [Equations of Mathematical Physics] / A.N. Tihonov, A.A. Samarskiy. — Moscow, 1976. — 544 p.
4. Vladimirov, V.S. Obshchie funktsii matematicheskoy fiziki [Generalized Functions in Mathematical Physics] / V.S. Vladimirov. — Moscow, 1979. — 320 p.
5. Pimenov, Yu.V. Tekhnicheskaya elektrodinamika [Technical Electrodynamics] / Yu.V. Pimenov, V.I. Volman, A.D. Muravtcov. — Moscow, 2002. — 536 p.
6. Dmitrieva, I. Problemy diagonalizatsii v klassicheskoy Maksvellovskoy teorii i ikh promyshlennoe primenenie [Diagonalization Problems in the Classical Maxwell Theory and Their Industrial Applications] / I. Dmitrieva // Proc. of the Intern. Scientific Conf. on the Econophysics, Complexity, etc. (ENEC 2008), Bucharest, Hyperion Univ., May 2008. – Bucharest, 2008. — Vol. 1. — pp. 11 — 23.
7. Dmitrieva, I. O konstructivnom reshenii sistemy uravneniy n-mernykh differentsial'nykh operatorov i ego primenenii v klassicheskoy Maksvellovskoy teorii [On the Constructive Solution of n-dimensional Differential Operator Equations' System and its Application to the Classical Maxwell Theory] / I. Dmitrieva // Proc. of the 6th Congress of Romanian Mathem., Bucharest, June 2007. — Bucharest, 2009. — Vol. 1. Scientific Contributions. — pp. 241 — 246.

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